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ENERGY DISTRIBUTION IN MODES IN THE WAKE OF A FINITE-LENGTH CYLINDER BEFORE AND AFTER TRANSITION

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The flow in the wake of a finite-length cylinder has been studied experimentally both before and after the transition to turbulence. This instability occurs at Reynolds numbers around 180–190. One end of the cylinder was fixed to the bottom of the test-section of a wind tunnel, whilst the other terminated in the open flow (free end). For these boundary conditions four main frequency modes within the wake can be identified. These are a centre-cell mode at the Strouhal frequency, end-cell modes with a frequency below the Strouhal frequency, a mode exhibiting the difference frequency between the centre-cell and end-cell modes, and a low-frequency mode (appearing only after transition to turbulence). In this work the energy content of these four modes has been determined throughout the wake, both before and after transition to turbulence. For three of the modes, the energy content is the same before and after transition, whereas the low-frequency mode exhibits energy two to four orders of magnitude greater after transition than before. Hence it is clear that the additional turbulence energy appearing in the wake after transition is located predominantly in this low-frequency mode. The appearance of this low-frequency mode is characterized by the simultaneous appearance of a peak in the power spectra of the velocity fluctuations centred about zero frequency (but with finite width). Consequently, the appearance of this zero frequency peak can be taken as the signature of the onset of turbulence. By considering the downstream growth rates of this low-frequency mode, evidence is presented which suggests that transition to turbulence may occur as a result of wake transition in the downstream central plane of the cylinder.

1. INTRODUCTION

THE FLOW OF A FLUID around a circular cylinder has long served as one of the archetypal systems used for the study of wake formation in bluff body flow. Although many hundreds of papers have been written on the flow structure of wakes, the mechanism by which the wake first becomes turbulent has remained an interesting scientific problem. The flow structure is governed by the Reynolds number $\mathrm{Re} = Ud/\nu$, where U is the free-stream flow velocity, d is the diameter of the cylinder and ν is the kinematic viscosity. The evolution of the wake up to $\mathrm{Re} \sim 180$ is now relatively well understood (Huerre & Monkewitz 1990; Williamson 1996a) but the secondary instability of the Karman vortex street, which for an infinitely long cylinder is known to occur at $\mathrm{Re} = 188{\cdot}5$ (Henderson & Barkley 1996), is still the subject of much discussion (Zhang *et al.* 1995; Williamson 1996b, 1992). This secondary instability is of some importance because it is at this stage that turbulence, in the form of stochastic fluctuations of the velocity field, is first observed globally in the wake.

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Recently, progress has been made on the nature of the laminar–turbulent transition† by Leweke & Provansal (1994), who investigated the flow through a circular ring. The use of a ring enabled them to replace the effects of finite-end boundary conditions with periodic end conditions and to illustrate that the transition to turbulence seems to be well characterized by the Ginzburg–Landau equation.

The presence of boundary conditions for a straight cylinder introduces complications that are now known to be of considerable importance (Williamson 1988). They influence the flow structure across the whole span of the cylinder, even in the case of large aspect ratios. Generally speaking, the flow structure can be crudely characterized into two main categories (Williamson 1989): parallel vortex shedding, where the vortex tubes are shed parallel to the cylinder, and oblique vortex shedding where they are shed at a finite angle. In practice, oblique vortex shedding is far more common than parallel shedding, which can only be induced by the use of specially developed techniques (Williamson 1989; Eisenlohr & Eckelman 1989; Hammache & Gharib 1991). The motivation for studying parallel shedding comes from the fact that it is believed to closely approximate the flow structure of an infinitely long cylinder. However, this is not the structure one would normally expect to observe in engineering or naturally occurring systems. For this reason, it is important that the transitional behaviour of flow structures, which are a consequence of the finite-aspect ratio of the body, are studied. It is in this context that the work reported here is presented.

Recent work by Williamson (1992, 1996b) suggests that natural transition to turbulence in cylinder wakes is a consequence of an amplification of vortex dislocations that are generated in the near wake of the cylinder. Experimentally, it is known that at the secondary instability of the vortex street a new spatial mode is observed. This is often referred to as mode-A and is characterized by the appearance of streamwise vortices with a wavelength of approximately four cylinder diameters in the spanwise direction. It is at this secondary instability that additional randomly occurring dislocations seem to appear along the span of the cylinder and, hence, turbulence is first observed. Vortex dislocations are common in finite-aspect-ratio cylinder wakes, where oblique shedding is observed, and are formed at the boundaries between cells of different vortex shedding frequency. They arise as a consequence of the nonlinearity of the flow and have a characteristic frequency equal to the difference of the two cellular frequencies. However, it should be noted that, typically, they exist prior to transition to turbulence but are localized at fixed positions along the span of the cylinder. This type of dislocation was termed "one-sided" by Williamson (1992). Williamson's picture would suggest that, at transition, additional "two-sided" dislocations occur along the span of the cylinder and are amplified as they propagate downstream.

In the work presented here, experiments were carried out on the wake of a finite-length cylinder placed in a boundary layer, with the other end terminating in open flow (free end). Such a situation is more typical of engineering applications where the presence of an excrescence alone causes transition in a flow. Although, at first sight, these boundary conditions seem quite different with respect to other work carried out on cylinder flows, in practice it was found (Stocks *et al.* 1996) that they gave rise to a rather simple and stable wake structure that is similar in form to wake structures observed in other studies.

The results of this study indicate that the wake first becomes globally turbulent due to an instability in the wake that amplifies low-frequency disturbances generated in the near-wake region. This amplification process gives rise to a zero-frequency peak in the power spectra of the velocity fluctuations. It is suggested that this mechanism may be consistent

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†Throughout this paper we use the term laminar–turbulent transition to simply refer to a transition from a time periodic to a time aperiodic state.

with Williamson's dislocation picture, in that it is the instability that selectively amplifies the low-frequency dislocations.

2. THE EXPERIMENT

The experimental set-up has been described in detail elsewhere (Stocks *et al.* 1996), but briefly consists of a circular cylinder, with a diameter, d of 1·6 mm and a length, l of 100 mm, mounted vertically in the test-section of an open-circuit wind tunnel. One end of the cylinder is secured to the base of the test-section and the other left as a free end. The flow velocity in the wake of the cylinder was measured using a hot-wire probe mounted on a computer controlled x–y–z traverse system. This enabled accurate positioning of the probe tip at a large number of pre-defined points. The velocity time-series at each point was digitized with 12-bit accuracy and stored on disk for later analysis. The wake produced by this simple cylinder arrangement is known (Stocks *et al.* 1996) to possess the features typical of finite-aspect-ratio cylinder flows, such as vortex dislocations and spanwise frequency cells. The role of the boundary conditions is to fix the position of the pre-transitional vortex dislocations and induce oblique vortex shedding.

3. THE WAKE STRUCTURE

Before going on to discuss the transitional behaviour in detail, it is important that the wake structure prior to transition be understood. As we will see, the spatial location of spanwise frequency cells and structure of the wake in general has an important bearing on the transition process. Figure 1 represents a spatial mapping of the frequency of the dominant spectral component in the spanwise, z, and downstream, x, directions; the plane is taken at $y/d = 0$, where y is the cross-stream co-ordinate. The colour represents directly the frequency of the dominant mode at each spatial location. The frequency of the dominant mode was obtained by calculating the power spectrum at each spatial location and finding the frequency of the largest (dominant) peak. The top of the cylinder corresponds to $z/d = 0$, and $z/d = 62{\cdot}5$ (not shown) to its base. Figure 1(a, b) represents two different Reynolds numbers, 158 and 189, thereby showing the wake structure prior to, and after, transition to turbulence.

Prior to transition, the wake has a rather simple structure that is now well understood. Directly behind the cylinder, the flow is split into three separate frequency cells: a dominant central cell that corresponds to the main vortex shedding mode (light green area), and two end cells which have lower shedding frequencies (turquoise area). Physically, this cellular structure arises through vortex dislocations at the cellular interfaces and is a direct consequence of the finite-end boundary conditions (König *et al.* 1992). We use the term "mode" to refer to oscillations with a different dominant frequency. In fact, the cellular modes observed in the wake are almost sinusoidal in time over a large area of the wake region. These cellular modes can be thought of as the primary linear modes generated in the near wake. All other frequencies observed in the spectra (prior to transition to turbulence) are linear combinations of the frequencies of these primary modes. However, for simplicity of terminology we also refer to other (nonprimary) frequency oscillations as modes. These will be defined in the text as we proceed. The Strouhal frequency of the central cell was measured to be 163 Hz, which corresponds to a Strouhal number ($S = fd/U$) of $S = 0{\cdot}180$. This is to be compared with the predicted value for parallel shedding (Williamson 1989) of 0·186. The discrepancy is due to the oblique nature of the shedding (this has been confirmed using measurement with two probes) which, using the formula (Williamson 1989) $S_{\mathrm{obl}} = S_{\mathrm{par}} \cos\theta$, is calculated to have an angle $\theta = 14°$.

Moving downstream, the red and brown areas are a result of an effective frequency doubling of the near-wake cellular frequencies. This occurs when alternately shed vortices are picked up equally by the hot wire. The fact that these regions do not extend into the near-wake region is a result of the orientation of the hot wire (which measured the x–z components of velocity) and its finite width. Also, as will be shown below, there is a tendency for higher frequencies to be attenuated more quickly as the flow convects downstream. Consequently, it would be expected that in the central plane, where there is an effective frequency doubling of the Strouhal frequency, that this frequency mode is attenuated downstream, leaving the original vortex shedding mode dominant.

The blue regions that emanate from the cellular interfaces are caused by the nonlinear mixing of the corresponding cellular modes; this gives rise to the generation of a mode at their difference frequency, which in this case is about 31 Hz. As this frequency is significantly smaller than the main vortex shedding frequency, it is found to be easily propagated down-stream whilst the higher frequencies are rapidly filtered out (Williamson 1992; Stocks *et al.* 1996). Consequently, it is this mode, generated by the vortex dislocations, that organizes the far-wake region and, as we will see, has important consequences for the wake structure after transition.

As mentioned previously, the cellular structure of the wake prior to transition is already well known. It is the investigation of the post-transitional wake structure that is of primary interest — this is illustrated in Figure 1(b). This colour map represents the same plane as in Figure 1(a) but now at a Reynolds number of 189. A comparison of Figure 1(a, b) indicates that the near-wake structure is largely unchanged. The frequencies of the cells themselves have increased, but the positioning and number of cells has remained unchanged. This is in stark contrast to the far-wake region that has developed a large dark blue region, positioned centrally in the wake that corresponds to a frequency of about 1 Hz. In fact, this new area does not arise from the generation of another harmonic mode. It is the result of a noisy low-frequency modulation of the velocity time-series and represents the main turbulent component in the wake after transition. This is demonstrated further in Figure 2, where plots (a) and (c) show two velocity time-series measured at the same spatial location, inside the turbulent dark blue region, but at the two different Reynolds numbers of 158 and 189. Clearly, the main vortex shedding mode has become unstable in (c). This is further borne out by their respective power spectra shown in Figure 2(b, d). After transition, a zero-frequency peak has developed in the broad-band background. The term "zero frequency peak" simply implies that the peak is centred about zero frequency and not that we are measuring the DC (mean flow) component which has been subtracted from the time series. The magnitude of this peak is not small and surpasses the magnitudes of the main vortex shedding modes. As we will show, it is this new peak that is the main signature of turbulence in our system. Such a peak has recently been observed in another cylinder flow experiment (Williamson 1996b), where large-scale, low-frequency fluctuations are observed to grow downstream of the cylinder. Evidence was presented that indicated that these low-frequency fluctuations were a result of vortex dislocations which spontaneously appeared after the wake had undergone its secondary transition to spatial mode-A. However, the energy content of this peak was not studied in detail. We suggest that the mechanism of transition observed in our system may be consistent with that proposed by Williamson.

Before moving on to discuss the nature of the turbulence, it is worth considering its spatial extent. Figure 1(b) indicates that the turbulent region is bounded on either side by the two "difference" modes generated at the interface of the cells. It was found, by consideration of the time-series data and power spectra, that these interfacial modes did not become turbulent, nor did the two end-cell modes. This indicates that only the dominant central vortex shedding mode loses stability and becomes turbulent at this Reynolds

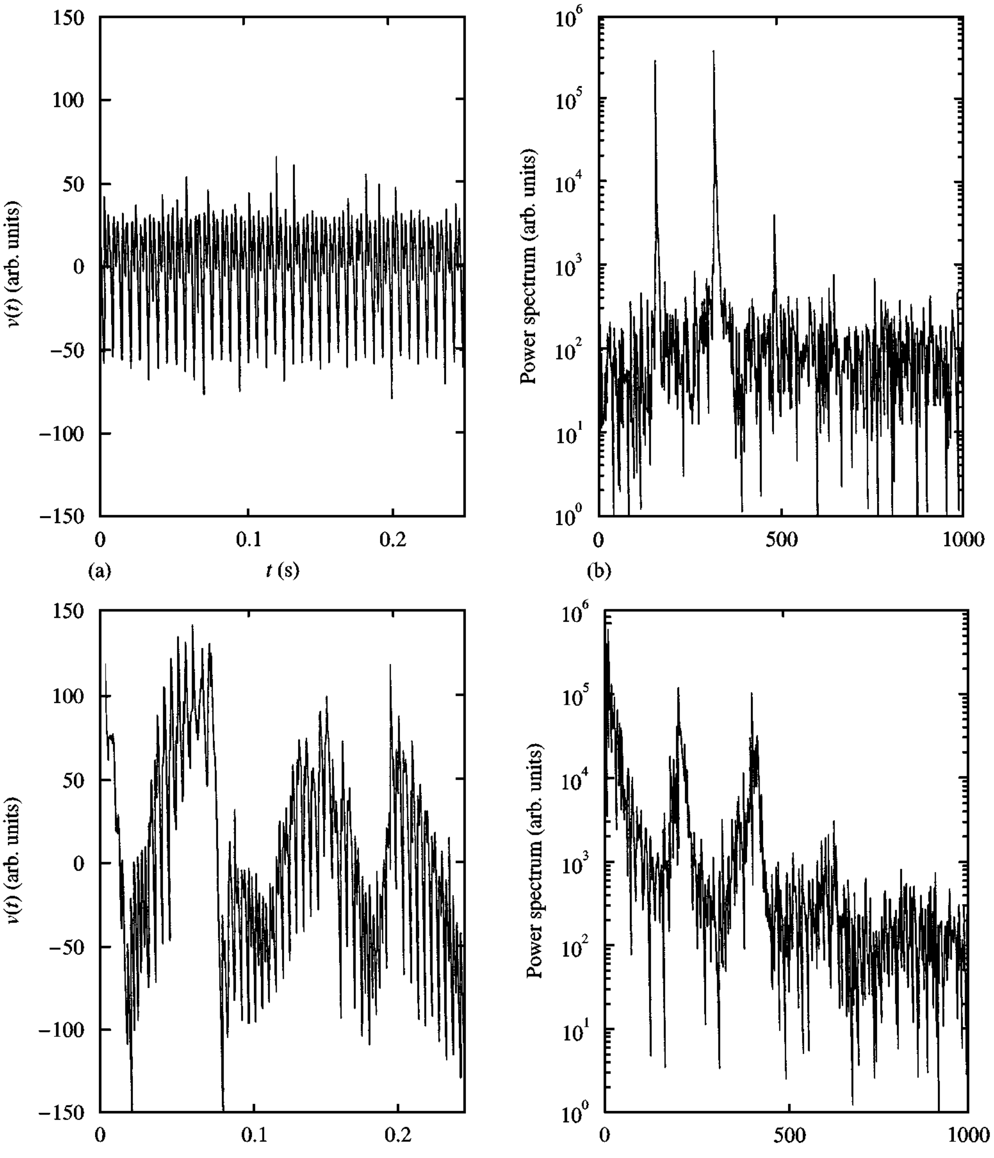

Figure 2. Velocity time series and their power spectra at $z/d = 26$, $x/d = 7{\cdot}5$, for (a, b) Re = 158 and (c, d) Re = 189.

number. This would tend to suggest that one may be able to model the lower frequency associated with the end cells by a rescaling of the Reynolds number based on the S − Re empirical formula (Williamson 1989). A sufficiently large increase in Reynolds number (approximately 20%) did result in the end cells becoming turbulent, but this effect has not yet been studied in depth.

4. ENERGY CONTENT OF THE MODES

Turning now to the evolution of the modes with downstream position, Figures 3 and 4 illustrate the variation of the average spanwise energy, $\langle E\rangle$, of each mode as a function of

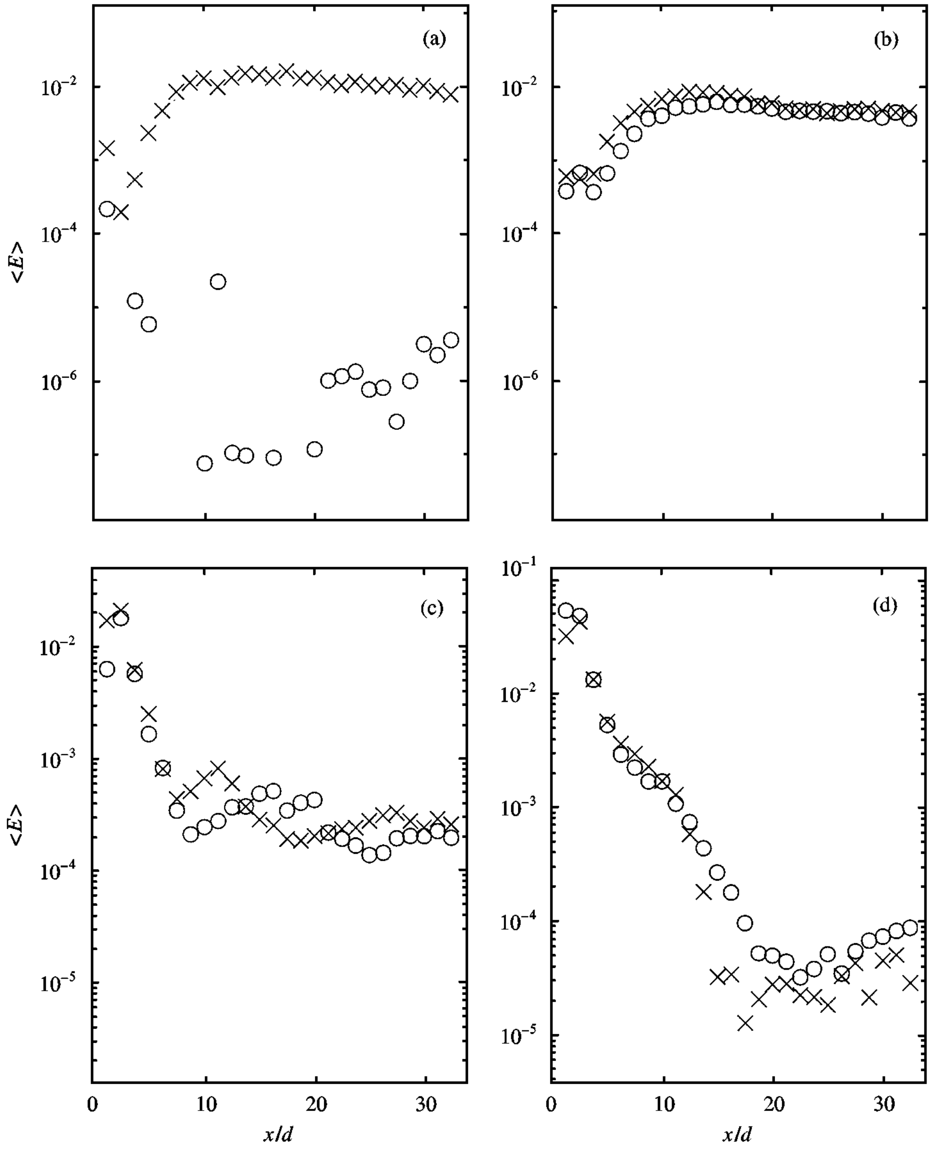

Figure 3. Average spanwise energy, $\langle E\rangle$, of modes as a function of downstream position in the centre-line plane $y/d = 0$, Four modes are shown: (a) the low-frequency mode; (b) the difference-frequency mode; (c) the end-cell mode; and (d) the centre-cell mode. Circles represent the energy in a mode at Re = 158. Crosses represent the energy in a mode at Re = 189.

downstream position, for two different planes in the wake of the cylinder. The average energy content of the modes was calculated by finding the energy of each mode from its Fourier component in the power spectrum, at a given spatial location, and then summing, for each mode in turn, along the span of the cylinder. This measure is equivalent to a time average of the square of the velocity and hence, experimentally, has units of $m^2 s^{-2}$. The energy of the turbulent mode was found by summing the lowest four adjacent frequency bins in the power spectrum. As this was the same number of frequency bins used to compute the energy of the harmonic modes, all the curves can be quantitatively compared.

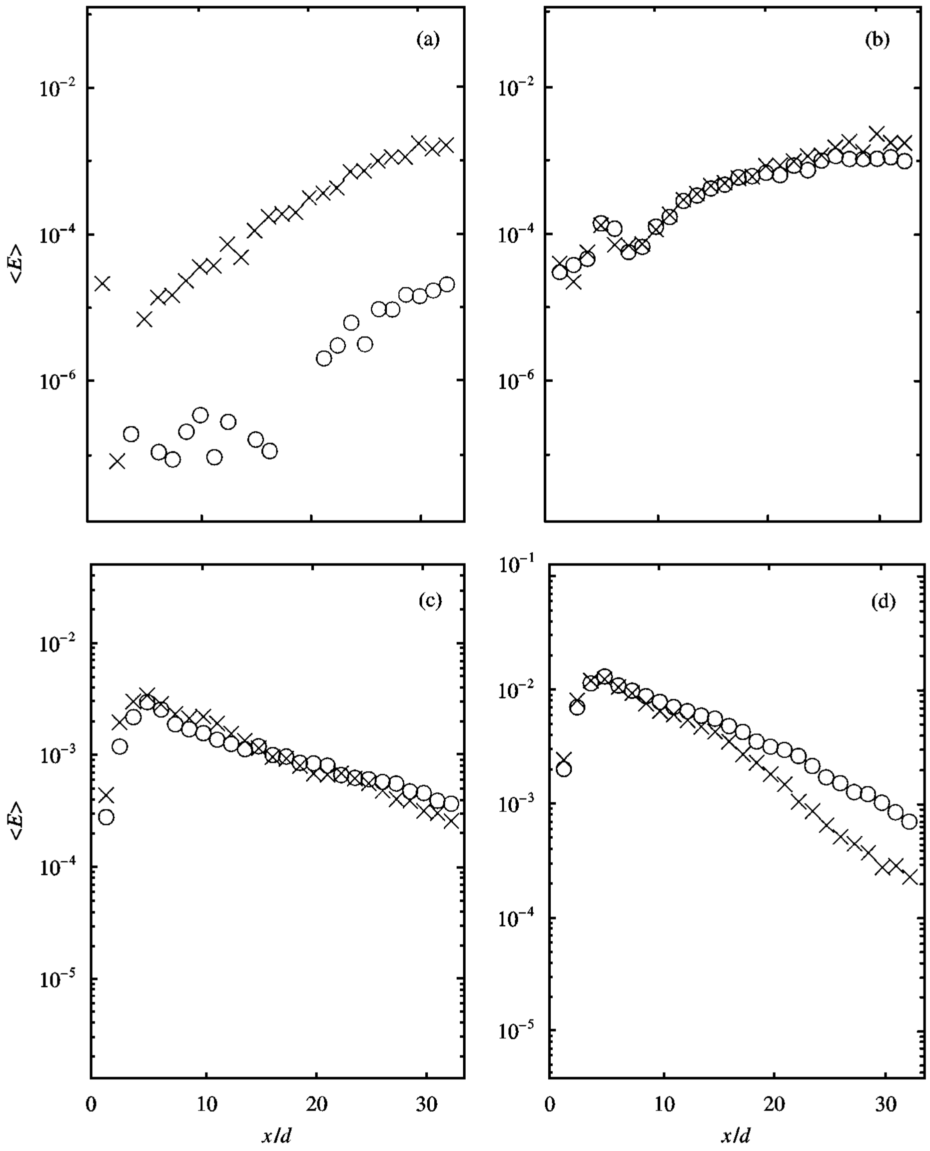

Figure 4. Total spanwise energy as a function of downstream position in the offset-plane at $y/d = 1{\cdot}25$. Symbols are as in Figure 3.

Figure 3 represents the energy in the centre-line plane $y/d = 0{\cdot}0$ which is directly downstream of the cylinder, whilst Figure 4 shows the energy in an offset plane at $y/d = 1{\cdot}25$. In this latter plane, the vortex shedding from one side of the cylinder alone is detected.

Looking at Figure 3(d) and 4(d), the variation in energy of the centre-cell mode can be seen. The energy of this mode, both before (circles) and after transition (crosses), decays with downstream position except for some growth initially in the near-wake region and for a levelling-off in energy in the centre-plane. Figure 3(c) and 4(c) show a similar variation for the end-cell modes. Hence these two modes, which exhibit the largest frequencies in the wake, appear to have very similar behaviour to each other.

Moving to the energy of the difference-frequency mode shown in Figures 3(b) and 4(b), here again the behaviour before and after transition is virtually unchanged. The energy rises downstream in the centre-plane to $x/d = 15$ and then decays slightly. Similarly, in the offset plane the energy rises, but at an ever decreasing rate, with the maximum position not found for the range of downstream positions taken in the experiment.

Hence, the energy of these three modes behaves in a similar way both before and after transition. This contrasts with the behaviour of the energy in the low-frequency (turbulent) mode, shown in Figures 3(a) and 4(a). In the centre-plane, the energy in this mode before transition decays in the near-wake region before rising slightly downstream. After transition, there is an initial fall, then rise in energy to a maximum at around $x/d = 15$, before a shallow fall off in energy. Similarly in the offset-plane this mode exhibits some fluctuation in energy, then a rise in energy, before transition; but a sharp fall, then rise, after transition with the rate of rise decreasing. Also the energy after transition in this mode is three to four orders of magnitude greater than that before transition in the centre-plane and some two orders of magnitude greater in the offset-plane. This indicates that prior to transition the low-frequency mode is not really present in the wake.

An important difference in the behaviour of the low-frequency mode, between the centre-plane and offset-plane, can however be noted. Before and after transition, the downstream growth rate of the low-frequency modes is similar in the offset-plane, whereas in the centre-plane the rates are very different. Indeed, in the centre-plane, there is a large qualitative change in the behaviour after transition. This would tend to suggest that, in the centre-plane, the wake itself undergoes some form of transition. Whereas, in the offset-plane, the increase in the magnitude after transition can be simply attributed to an increased energy in the near wake of the cylinder that is then propagated downstream.

Furthermore, after transition, the energy of the low-frequency mode in the centre-plane is much larger (as $x/d = 10$ it is three orders of magnitude larger) than in the offset-plane. Consequently, it is this increase in turbulent energy in the centre-plane that accounts for the greater part of the total energy in the wake.

5. DISCUSSION

From the behaviour of the energy in the four modes we can postulate that these effects can be divided into two related groups. The energies in the centre-cell and end-cell modes are related, as are the energies in the difference-frequency mode and the low-frequency mode. The centre-cell and end-cell modes both decay rapidly with downstream position, with a faster decay observed in the central plane. This behaviour is qualitatively the same at both Reynolds numbers implying that they are not affected by the transition itself.

This is to be contrasted to the behaviour of the difference- and low-frequency modes. Before transition, the turbulence (low-frequency) energy in the wake is a negligible fraction of the total wake energy at all positions downstream. After transition, a rapid increase in energy is observed as one moves downstream. At its maximum, the turbulent mode has grown by five orders of magnitude, compared to the pre-transitional case. This rapid growth, or amplification, results in the whole far-wake region becoming turbulent. Effectively, after transition, the turbulent fluctuations, already present in the near-wake region, become present in the rest of the wake. Also, after transition, the low-frequency mode shows the same downstream dependence as the difference mode. This indicates that the wake always selectively amplifies perturbations at the difference frequency, but the increase in Reynolds number extends this amplification mechanism to lower frequencies.

It is important to note that the significant qualitative change in the behaviour of the low-frequency mode in the centre plane before and after transition is indicative of a transition in the wake itself and not simply an increase in the turbulence energy in the near wake. If the increase in turbulence was solely due to an increase in the fluctuations in the near wake, then one would not expect different growth rates before and after transition. However, this argument only applies to the central wake region. Consequently, this would tend to suggest that turbulence is driven by the properties of the wake in the centre plane of the cylinder.

If one tries to interpret these results within the framework of Williamson's vortex dislocations picture, then one would speculate that it is the secondary instability of the vortex street, giving rise to transition to mode-A, that modifies the properties of the wake, but only in the central plane of the wake. This transition results in the central plane region becoming unstable to low-frequency fluctuations. Hence, these fluctuations, which already exist in the near wake region prior to transition, are selectively amplified and give rise to large scale vortex dislocations. Whilst this is obviously a simplified view of what is a complicated spatiotemporal effect, we believe it may encompass the main features of the transition process.

Finally, it can be seen that the low-frequency mode is increasing in energy with downstream position before transition in both planes. This suggests that at sufficient distances downstream, the turbulence energy may become greater than or comparable with the energy in the other modes. It is suspected that this behaviour is associated with a much weaker instability, where the growth rate is sufficiently small that its effects are not observed over the greater part of the near wake. A similar type of behaviour has been noted previously by Cimbala *et al.* (1993)† who, based on an inviscid linear stability analysis, showed that finite downstream growth rates would occur down to zero frequency. However, the growth rates associated with these low frequencies are predicted to be small, and whilst they possibly account for the behaviour of the turbulent mode in the off-centre plane, and in the central plane prior to transition, they do not predict the rapid amplification observed in the central plane after transition.

6. CONCLUSIONS

In conclusion, we would propose that these results, taken with the mechanism proposed by Williamson (1992, 1996b), suggest that the occurrence of turbulence is due to the selective amplification of low- frequency fluctuations in the near wake. Our results also indicate that this amplification mechanism arises due to a transition in the wake itself. However, it appears that the wake properties are only strongly affected by this transition in the central downstream plane of the cylinder. It is in this central plane that the greatest amplification of fluctuations is observed and, hence, it would appear to be this region that nucelates the generation of turbulence. We postulate that it is the transition to mode-A that actually causes the modification of the wake properties and hence to the appearance of the amplification mechanism. This would suggest that naturally occurring turbulence can only be observed after the formation of this mode.

In addition, amplification appears to occur down to zero frequency; this would imply that any imperfection in the flow along the affected region of the cylinder could be amplified. For example, small local variations in the uniformity of the flow could be amplified, no matter how small.

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†See also Williamson & Prasad (1993).

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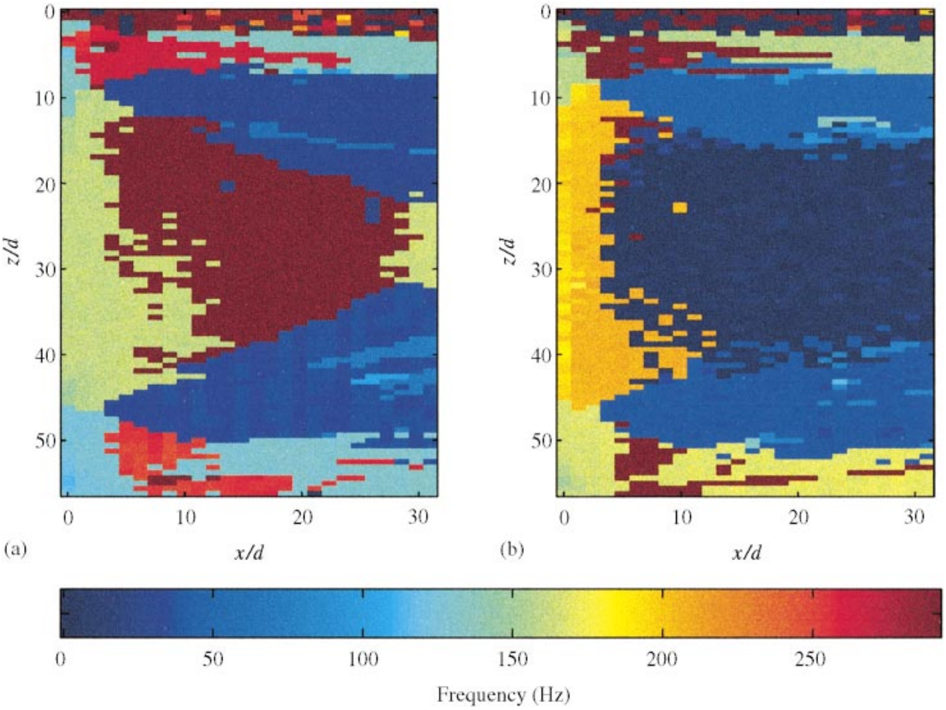

Figure 1. Colour maps of the spatial distribution of the dominant spectral frequency for (a) RTe = 158 and (b) Re = 189. The plane is taken on the central plane of the cylinder at $y/d = 0$.